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The rotation of the Uranian system

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The rotation of Uranus is examined for clues as to the origin of the Solar System. Both theories based on the formation of planets through the accretion of small planetesimals, and theories based on the formation of giant gaseous protoplanets through a gravitational instability in the primitive solar nebula allow for qualitative explanations of the large tilt of Uranus's equator to the orbital plane, and the fact that its satellites lie in the equatorial plane. Models of the planetary interior show that the mass ratio of ice-forming materials to rock in Uranus's interior must be more than about three if the rotation period is about 16 h. Such a large ratio seems to exclude those accretional theories that require most of the nebular gas to be heated to relatively high temperatures before being accreted into the planet.

Introduction

The question of how the Solar System formed is one of the oldest questions in planetary science. The fact that it is still unanswered, indicates that it is also one of the most difficult. Part of the difficulty arises from the complexity of the processes involved, and part from the interplay among a number of different and competing effects. Theoretical studies must, of necessity, content themselves with making a number of simplifying assumptions, and so it is always desirable to test the results of these assumptions by comparison with observations. Uranus offers a number of opportunities for such comparisons. Before detailing some of these comparisons, however, it is useful to review some of the ideas of modern cosmogonic theory.

Although there are quite a number of theories in the literature, most can be fitted into one of two broad classes. The first class consists of accretional theories. These assume that the Sun was at one time surrounded by a gaseous nebula of some 10^{-1} solar masses. There are several ways in which such a nebula might have been acquired, and the reader is referred to the literature for details (Prentice 1978; Safronov 1972; Alfvén & Arrhenius 1975). The temperature at any point in the nebula was determined by its distance from the Sun. The temperature, in turn determined what the chemical composition of the nebular material was, and which components were solid, (Grossman 1972; Lewis 1972). For convenience I shall use the term 'rock' for any material, such as iron or magnesium silicate, that is solid under most of the conditions found in the nebula, and 'ice' for materials like water and methane that can be solid under some conditions, and gaseous under others. Near the Sun only the rocky materials were solid, and the accretion of such material gave rise to the terrestrial planets. Further from the Sun the ices were solid as well, and a greater reservoir of material was available for accretion. When the mass of the accreting protoplanet reached a large enough value (typically some ten Earth masses), a hydrodynamic instability was induced, and some portion of the nebular gas collapsed onto the embryo (Mizuno 1980; Bodenheimer & Pollack 1983), forming the giant planets.

A second class of theories is based on the idea of a solar nebula with a mass of the order of solar mass. Owing to gravitational instabilities, portions of the nebula collapse to form Jovian-sized protoplanets. These giant gaseous protoplanets (g.g.ps) then lose mass and evolve into the planets (both terrestrial and giant) that we see today (McCrea 1978; Cameron 1978*a*, *b*). Another theory that also results in g.g.ps is that of Woolfson (1978) who envisages a filament of gas being drawn by the Sun from a passing protostar. Such a filament will be unstable and will also break up into several g.g.ps. Clearly it would be desirable to find some way to distinguish between the accretion and g.g.p. scenarios observationally.

Rotation of Uranus: direction

The rotation of Uranus holds some important clues for the resolution of this problem. In the first place there is the tilt of the rotation axis. Uranus is unusual in that its equator is tilted 98° to the plane of its orbit. Can this fact be understood within the context of either the accretion or g.g.p. scenarios? For the accretion scenario the analysis, due to Safronov (1972), is fairly straightforward. Suppose that a planetary embryo has an angular momentum described by a vector $\boldsymbol{H}$ which is the sum of a normal component $\boldsymbol{H}_0$, and a random component $\boldsymbol{H}_1$. This random part is due to the infall of planetesimals. The planetesimal size distribution is given by $n(m,t) = c(t)\, m^{-q}$, where n is the number of planetesimals at time t with masses between m and $m+\mathrm{d}m$, q is some constant, and $c(t)$ is a time-varying proportionality constant. If G is Newton's constant, M is the mass of the protoplanetary embryo, r is its radius, m_1 is the mass of the largest accreted planetesimal, and θ is a constant that describes the gravitational mixing of planetesimal orbits, Safronov shows that

$$H_1 = M[0.3(1+0.5/\theta)\, Gm_1\, r(2-q)/(3-q)]^{\frac{1}{2}}. \tag{1}$$

If the angle between $\boldsymbol{H}_1$ and $\boldsymbol{H}_0$ is ϕ, and the angle of inclination of the rotation axis is ϵ, then $\boldsymbol{H}_1 \sin\phi = \boldsymbol{H} \sin\epsilon$. The mean value of ϕ is given by $\langle \sin^2\phi \rangle = \frac{2}{3}$, and we have

$$m_1/M = [(3-q)/(2-q)]\, [5\sin^2\epsilon/(1+1/2\theta)]\, H^2/GM^3 r. \tag{2}$$

For the case of Uranus $H_1 > H_0$, and so the analysis must be modified somewhat. Since $\langle \sin\phi \rangle = \frac{1}{4}\pi$, Safronov takes $\sin\epsilon = \pi H_1/4H$, which gives m_1/M in the range 0.02–0.08. For Saturn, the analysis gives m_1/M in the range 0.01–0.06. These values of m_1/M are not inconsistent with the expectations of models of the accretion process (Greenberg 1979). Thus, Uranus, in this sense, is not very unusual when viewed from the standpoint of the accretion scenario.

G.g.p. scenarios have somewhat more difficulty in explaining the tilt of Uranus. The g.g.ps would be expected to partake of the general rotation of the nebula and thus have relatively small axial tilts. On the other hand, if the nebula was massive enough, many g.g.ps would be expected, and mutual interactions may result in one or more planets with an exceptionally large tilt. No detailed study of this question has yet been made. It is interesting to note that in this regard the theory of McCrea (1978) behaves as an accretional theory, since in this theory there are individual turbulent eddies in the nebula acting as independent 'floccules' which can collide and grow. In this picture, a g.g.p. is formed from a combination of some 10^2 floccules, and from the analysis of Safronov (1972) it can be seen that g.g.ps with large axial tilts will not be unusual.

There is one other bit of information with regard to the axial tilt of Uranus, and that is that the plane of the satellite orbits is very nearly in the equatorial plane of Uranus. Singer (1975) has argued that since the satellite orbits would not follow the equatorial plane for a rapid tilting of the planet, and since a slow tilting is unlikely, the satellites must have been formed after the tilting took place. Possibly, the very process that caused the tilt formed the satellites as well. Thus in the picture of Safronov, the tilt is caused by the impact of a large planetesimal of a small fraction of the planetary mass in the late stages of accretion. This impact could also have generated a circumplanetary disc from which a satellite system could have formed. Such a system would, in fact, lie in the equatorial plane of the planet (Ruskol 1972; Safronov 1972). Once again the accretion scenario seems to provide an acceptable framework for explaining the rotation of the Uranus system. The g.g.p. scenario too can provide such a framework. Here satellite systems are seen as forming in a subdisc produced by the contraction of the g.g.p. Once the axial tilt of the g.g.p. has been determined, the plane of the regular satellites is, to a large extent, fixed. In this picture too, however, the tilting must occur before the formation of the subdisc. We see then that the direction of the angular momentum vector, although containing much information about the origin of the Solar System, cannot, as yet, help us decide between the accretion and g.g.p. scenarios.

Rotation of Uranus: magnitude

A second source of information is the magnitude of the rotation vector. Here I am referring not to the rotation rate as a datum in itself, although that too contains valuable information. Rather I am referring to the moment of inertia of Uranus, which can be deduced from the flattening of the planet if the rotation rate is known. Here the reasoning is somewhat indirect, and depends on the composition one would expect for a given cosmogonic scenario. Thus in the case of a g.g.p. scenario, one has a portion of the nebula collapse, and therefore the initial composition of the g.g.p. should be solar. In particular, the ratio of ice to rock (i/r) should be appropriate to solar composition material which has cooled to the temperatures and pressures found in a g.g.p. interior. The difficulty comes from the fact that Uranus and Neptune (and to a lesser extent Jupiter and Saturn) are clearly not of solar composition, but have an excess of ice and rock relative to hydrogen and helium. Some loss mechanism must therefore be postulated to remove the hydrogen and helium, and this mechanism may alter the value of i/r as well.

The most detailed studies of the evolution of isolated g.g.ps are those of DeCampli & Cameron (1979). They found that the interior temperatures reach about 2000 K after which hydrodynamic collapse occurs due to hydrogen dissociation near the centre. The effective temperatures of these objects is low, however, being in the 30–100 K range. There may therefore be a cold trap in the outer layers which would prevent some of the ice-forming materials from passing back into the nebula. This would allow a means of enhancing the ices (and rock) with respect to the hydrogen and helium. If there were some mixing between the g.g.p. and the nebula, with a net passage of material out of the g.g.p., only those materials able to pass through the cold trap would be removed. Podolak & Reynolds (1984) have applied the criteria for chemical equilibrium to the models of DeCampli & Cameron (1979) and found that for the conditions of pressure and temperature computed, N_2 is the dominant nitrogen-bearing species above about 800 K, and CO is the dominant carbon-bearing species above 1600 K. Such temperatures are reached in the protoplanet several tens of thousands of years before the final collapse. Since most of the protoplanet is convecting with speeds of several metres per second (DeCampli & Cameron 1979), most of the nitrogen and carbon will be processed through the high-temperature region and converted into N_2 and CO. Although the re-conversion of N_2 into NH_3 at lower temperatures is kinetically prohibited (Lewis & Prinn 1980; Norris 1980), the re-conversion of CO to hydrocarbons can proceed quite rapidly at these pressures. Based on data from shock wave experiments (Bar-Nun & Shaviv 1975), Bar-Nun has found a formal rate constant for the reaction $CO+3H_2 \rightarrow H_2O+CH_4$ of $d[CO]/dt = k[CO]\ [H_2]^3$ where $k = 1.1 \times 10^{-51} \exp(-E_a/RT)\ cm^9\ mol^{-3}\ s^{-1}$, $E_a = 23000$ cal†, R is the gas constant, and T the temperature (Bar-Nun & Podolak 1984). This rate constant is meaningful in the 10^5 Pa and 10^3 K regions of pressure and temperature. In a g.g.p. with a temperature of 777.5 K at 0.11×10^5 Pa we have $d[CO]/[CO] = 4.2 \times 10^{-4}\ s^{-1}$, so that relatively complete conversion will take place on the timescale of an hour. This is typical of the timescales for conversion in all of the model g.g.ps computed by DeCampli & Cameron (1979). The ices in g.g.ps will therefore consist of mostly H_2O, N_2 and CH_4. The expected value of i/r in the present-day Uranus will depend on the temperature of the cold trap. If it is low enough to hold N_2, then all other volatiles of consequence (other than H_2 and He) will be trapped. We will then have $i/r \approx 3.5$ (Cameron 1981). If N_2 escapes, but not CH_4, this ratio will not be much affected, but if CH_4 escapes as well, the ratio will drop to *ca.* 2. Thus values of i/r between 2 and 3.5 are consistent with a g.g.p. scenario.

Handbury & Williams (1975) have suggested that the sedimentation of ice rock grains towards the centre of the g.g.p. would release enough gravitational energy to drive off the hydrogen and helium through the Jeans escape mechanism. In this case none of the ices will escape, as their molecular mass would be too high, and $i/r \approx 3.5$ is expected. Similarly, in the theory of McCrea (1978), the spin-up of the g.g.p. upon contraction causes it to shed material at the equator. Here there is a possibility of differentiation between grains of ice and rock, since the latter, being denser, will sediment more quickly, and those grains that are lost will have a greater fraction of ice in them. In this case an i/r less than 3.5 is possible, although this case has yet to be checked in detail.

The accretional scenario gives a rather different prediction. Solar material at high temperature will contain the ice forming elements as N_2, CO and H_2O, while at lower temperatures the equilibrium composition will be H_2O, NH_3 and CH_4. As Lewis & Prinn (1980) have shown, a circulation of material in the nebula will process most of the material through the high-temperature region. Upon its return to low temperatures, however, the material will be kinetically quenched, and remain in its high-temperature composition. At the temperatures expected in the vicinity of Uranus (50 K), both N_2 and CO will be gases, and will not be accreted. Since much of the oxygen is tied up in CO, the H_2O abundance is much reduced, and $i/r \approx 0.5$ is expected. What in fact is i/r for Uranus?

Results of modelling

Podolak & Reynolds (1981, 1984) have run models of Uranus and Neptune to deduce i/r. The models are of two types: two-shell and three-shell models. The two-shell models have a core of rock surrounded by an envelope of H_2 and He in solar proportions. Mixed into the

† 1 cal = 4.184 J.

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envelope is some mass of H_2O, CH_4, and NH_3 in the solar ratio to reach other. The envelope is assumed to be convecting, so that the temperature profile is adiabatic. The masses of the rock and ice are varied until a fit is achieved to the density of the planet and to J_2, the quadrupole moment of the gravitational field. The temperature is taken to be 75 K at 10^5 Pa in agreement with the most recent determinations (Gautier & Courtin 1979). The three-shell models are identical except that they have an additional ice shell between the envelope and the core. An additional constraint on the model is J_4, the 16-pole moment of the gravitational potential. Clearly the rotational period must be known in order to compute J_2 and J_4 for a given model, and this is where the rotation of Uranus comes in. Unfortunately, the rotation period has not yet been fixed unambiguously. Most of the recent measurements point to a period of about 16 h, this value being derived from the Doppler tilt of spectral lines (Brown & Goody 1980) and from the correlation of the planet's shape and J_2 (Franklin *et al.* 1980; Elliot *et al.* 1981). There is, however, a photometric determination which gives a period of nearly 24 h (Smith & Slavky 1979), as well as spectral measurements which also fall in this region (Belton *et al.* 1980). Thus both values must be considered possible. In figure 1 the results of the modelling

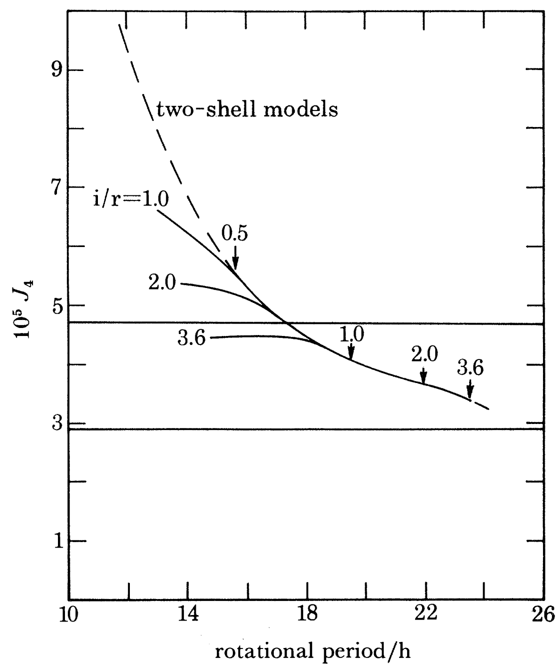

FIGURE 1. The rotational period that provides a match to J_2 against J_4 for that period for two-shell (– – –) and three-shell (——) models of Uranus. The numbers above the arrows show i/r for two-shell models at those positions. Horizontal lines show the uncertainty in the observed value of J_4.

effort are shown. The two horizontal lines show the limits on J_4, determined from the precession of the orbits of the Uranian rings (Nicholson, personal communication), while the curves show the trajectories of models which were given the period required to match J_2 (abscissa), and the J_4 appropriate to that period (ordinate). The dashed curve is for two-shell models, while the solid curves are for three-shell models with total i/r equal to 1.0, 2.0 and 3.5. In all cases the enhancement of ice in the envelope increases as one moves towards the lower right. The arrows indicate the value of i/r for two-shell models along the trajectory.

We see, first of all, that a value of i/r = 0.5 is too low to fit the observed J_4 for any model or rotational period. Indeed i/r must be greater than *ca.* 0.7 for a two-shell model for any rotational period. In addition, the two shell model is compatible only with periods longer than about 17.5 h. A 16 h period requires i/r to be greater than about three, with most of the ice in the intermediate shell. A period near 24 h again requires i/r of the order 3.6, so that in any event, the models of Uranus seem to rule out an i/r as low as 0.5. This does not mean that the accretion scenario is to be discarded, however. Possibly the reaction rates estimated by Lewis & Prinn (1980) need revision, or perhaps the temperatures at the place of Uranus's formation were much lower than 50 K so that CO would also be accreted as a solid. Thus although its large tilt is perhaps the most obvious feature of Uranus's rotation, it is through the magnitude of that rotation that the planet reveals its composition, and thus helps to set important constraints on cosmogonic theories.

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